An Iterative Error Patterns Library Formation Method for the Decoding of Product Codes

Xunhuan Ren
Dept. of Infocommunication Technologies
Belarusian State University of Informatics and Radioelectronics
Minsk, Belarus
renxunhuan@bsuir.by

Jun Ma
Dept. of Infocommunication Technologies
Belarusian State University of Informatics and Radioelectronics
Minsk, Belarus
majun1313@hotmail.com

Valery Konopelko
Dept. of Infocommunication Technologies
Belarusian State University of Informatics and Radioelectronics
Minsk, Belarus
volos@bsuir.by

Victor Tsviatkou
Dept. of Infocommunication Technologies
Belarusian State University of Informatics and Radioelectronics
Minsk, Belarus
vtsvet@bsuir.by

Abstract. **Product codes are preferred in high data rate wireless communication systems to achieve good error performance. In this paper, the problem of two-dimensional syndrome-norm decoding of product codes based on a library of error patterns is considered. In product coding, sequence code is first transformed into a code matrix, and then the row and column check code are calculated. In the decoder, the error position of the two-dimensional can be obtained by the operations that first calculate the syndromes and norms, then match with the error patterns in the existing library. The error pattern library is stored in the memory and generated by the subset of the error pattern. This paper proposed a mathematical model for fast generating a library based on the iterative expansion of the error patterns, which makes it possible to shorten the computational complexity in comparison with the known approaches.**

Keywords: **product codes, error-correcting coding, norm, syndromic-norm decoding, library of error patterns**

I. Introduction

Error correcting codes are used in many places, wherever there is the possibility of errors during transmission [1]. In Error control coding(ECC), parity check bits are calculated based on the input data. The input data and parity check bits are transmitted across a noisy channel. In the receiver, an ECC decoder is used to detect or correct the errors induced during the transmission. The number of parity bits depends upon the number of information bits. At present, the most successful coding schemes are turbo codes and low-density parity-check codes, since their excellent capability, closely to the Shannon limit. Under some specific requirement (typically, code-rates near to the unity and low error rates required), product codes may turn into competitive. Product codes [2, 3], which can be easily realized by concatenating simple component codes, have a good protection capability against both random and burst errors. Product codes, whose component codes are Hamming or extended Hamming product codes (BCH product codes), BCH product codes can be constructed to improve the error correction capability, but a more complex decoding process is required. To reduce the decoding complexity of BCH product codes based on the three-stage scheme[4], a syndrome-norm method based on library matching of error patterns is proposed in [5]. The syndrome-norm method is a decoding method. It can fast match the target pattern from the pattern library by using the norm, which is calculated based on the result of the syndrome vector, can uniquely determine the base pattern and the corresponding set of error patterns and then applied a predefine correction method to correct the errors. The library of error patterns consists of some basic patterns that delegate a set of error patterns with a common characteristic. If one pattern can be transformed into another pattern by exchanging internal two-row or two-column, then we consider that these two patterns share the same characteristics. The decoding ability of this method is wholly dependent on the error pattern library. Therefore, one of the limitations of this method is that the formation of the matching library is very time-consuming, so it is tough to form a pattern with a lot of errors.

Pattern Recognition and Information Processing (PRIP'2021) : Proceedings of the 15th International Conference, 21–24 Sept. 2021, Minsk, Belarus. – Minsk : UIIP NASB, 2021. – 246 p. – ISBN 978-985-7198-07-8.

In the recent ten years, two library formation methods have been proposed to overcome the problem mentioned above. To simplify the problem, all the patterns are referred as matrices, and in each matrix, each error position is marked as one. Then, it assumed that the maximum number of errors in a certain pattern is not above the square root of the total number of elements of it. A method based on rank has been proposed in [6]. This method first enumerates all the possibilities of error patterns in a given size. This pattern set is named as original error pattern set. Then it compares all these patterns to selects some patterns with distinct feature vector to adding to the library. The feature vector used in this method including the rank of the matrix. Another formation method is based on sub-classified, which proposed in [7]. It tends to first category the original patterns set into several subset according to some criterion then conduct the comparing them in a smaller set rather than directly compare them with each other. This method saves more time when comparing with the rank method. However, the sub-classified method still needs to generate all possible error patterns as the original pattern, which may cost a lot of time. As a result, it is necessary to develop a new library formation method that is not needed to list all the error patterns. In this paper, an iterative formation method has been proposed.

II. Basic Concepts

A. Product codes

Product codes are widely used to correct errors in data transmission and storage systems. They are formed in the verification codes $C_1(n_1, k_1, d_1)$ and $C_2(n_2, k_2, d_2)$ for the rows and columns of the source code matrix $n_1 \times n_2$, respectively, where n is the length of the code; k is the number of information symbols. The random-error-detecting and random-error-correcting capabilities of code are determined by its minimum distance $d_{\min}$, if the component codes C_1 and C_2 have minimum Hamming distances d_1 and d_2, accordingly, then the minimum Hamming distance of the two-dimensional product codes C_{pc} is the product d_1 and d_2 ($d_{\text{pc}} = d_1 \times d_2$), and at the same time $d_{\text{pc}} \geq 2t+1$, t is the multiplicity of corrected errors [8], which greatly increases the error correction capability. Concept of two-dimensional product (iterated) codes shows as Fig. 1.

The simplest two-dimensional product codes are single parity check (SPC) product codes, assured to correct only one error by inverting the intersection bit in the erroneous row and column [8, 9].

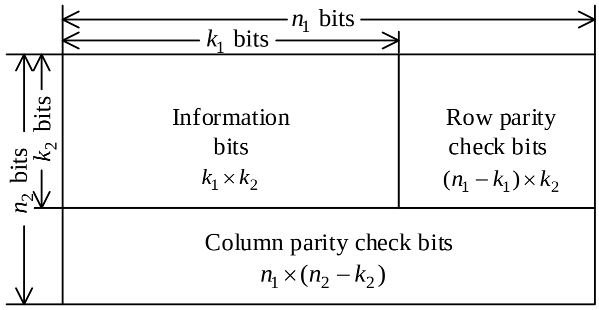

Fig. 1. Concept of two-dimensional product codes

Multidimensional SPC product codes can be constructed to improve the error correction capability, but a more complex decoding process is demanded [10].

B. Syndrome-Norm Decoding For Product codes

In the scheme of syndrome-norm decoding of product codes [4] , the syndromes and norms of rows and columns are first calculated. Then, these two parameters can match with a unique error pattern in the library. But before match operation, it is useful to shorten the search area by calculating the rough number of the error. After the matching operation, we will correct the error by a specified corrected method. The diagram of the whole decoding process is presented in Fig. 2.

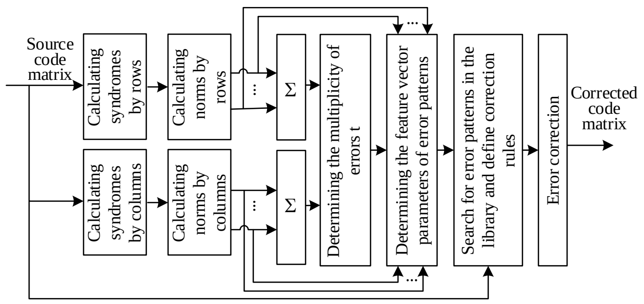

Fig. 2. Diagram of syndromic-norm decoding of product codes

C. Mathematical Condition for Library

The matching library consists of distinct selected error patterns. The selected error pattern is represented by $\{M_R(n)\}_{\left(n=\overline{1,N(t)}\right)}$, which should satisfy the following mathematical condition:

$$M_R(n) = \left\| m_R(n,i,j) \right\|_{\left(i=\overline{1,t},\, j=\overline{1,t}\right)}, \tag{1}$$

$$\sum_{i=1}^{t} \sum_{j=1}^{t} m_R(n,i,j) = t \,, \tag{2}$$

$$\neg\exists n_1 \neg\exists n_2 \left(M_R(n_1)\right) = f_{RT}\left(f_{CT}\left(M_R(n_2), j_c\right), i_r\right) \quad \text{when } j_c = \overline{1,T},\ i_r = \overline{1,T} \tag{3}$$

Pattern Recognition and Information Processing (PRIP'2021) : Proceedings of the 15th International Conference, 21–24 Sept. 2021, Minsk, Belarus. – Minsk : UIIP NASB, 2021. – 246 p. – ISBN 978-985-7198-07-8.

where $N(t)$ – total number of all selected error patterns; f_{RT} – functions for exchange two rows; f_{CT} – functions for exchange two columns; T – total number of possible exchanges in row or column, $T = C_t^2$; i_r , j_c – index of exchange of rows or columns.

III. PROPOSED METHOD

The proposed method is based on the fact that all the patterns with *n* errors can be obtained by extending one error from the patterns with $n-1$ errors. So there is only require to select the representative pattern from a small set rather than all the patterns. Therefore, the proposed method is an iterative method. The mathematic model of the proposed method is presented in following.

$$\left\{M_R(n,t)\right\}_{n=\overline{1,N(t)}} = \left\{M_x(n_x,t)\right\}_{n_x=\overline{1,N_X(t)}} \cup \left\{M_x(n_x,t)^T\right\}_{n_x(t)=\overline{1,N_x(t)}} \quad (4)$$

$$\left\{M_x(n_x,t)\right\}_{n_x=\overline{1,N_X(t)}} = f_M\left(\left\{M_d(n_d,t), p_{Rank}(n_d,t), P_{RC}(n_d,t), P_{Ws}(n_d,t)\right\}_{n_d=\overline{1,N_d(t)}}\right),$$

$$\left\{M_d(n_d,t)\right\}_{n_d=\overline{1,N_d(t)}} = f_{add}\left(\left\{M_x(n_x,t-1)\right\}_{n_x=\overline{1,N_X(t-1)}}\right),$$

$$N_d(t) = N_X(t-1)\cdot(t^2-t+1),$$

$$M_x(n_x,1) = m_x(n_x,1) = 1 \text{ when } N_x(1)=1,$$

$$\neg\exists n_1 \neg\exists n_2 \left(M_R(n_1) = \left(M_R(n_2)^T\right)\right) \text{ when } n_1 = \overline{1,N(t)},$$

$$n_2 = \overline{1,N(t)}, \quad (5)$$

where $M_x(n_x,t)$ – base error patterns at the *t*-th iteration, $M_x(n_x,t) = \left\|m_x(n_x,\mathrm{t},i,j)\right\|_{\left(i=\overline{1,t},\, j=\overline{1,t}\right)}$; $M_d(n_d,t)$ – extended error patterns that derived from the base error patterns; f_{add} – the function that expanding the error pattern and adding one single element.

$$P_{RC} = f_{sort}\left(p_{RE}, p_{CE}\right). \quad (6)$$

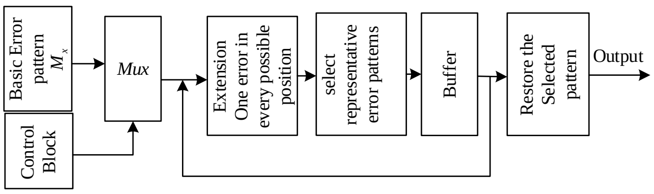

Fig. 3. Block diagram of error pattern generation method based |on the iterative extension

Fig. 3 shows the block diagram of the proposed method, it can be seen that a new set of error patterns can be obtained based on the former set of error patterns, go through a series of expanding operations, and selecting operations. By contrast with other generating methods, the search range of representative error patterns of the proposed is less, which may reduce the time for creating the whole library.

IV. EXPERIMENT AND RESULTS

The comparison of the computational speed is conducted on the basis of the implementation of three different formation methods. The platform of the experiment environment in Matlab under the windows 10 operating system. The average execution time for generating the error patterns library used by the different methods is summarized in Table I.

TABLE I. AVERAGE TIME FOR GENERATING ERROR PATTERN LIBRARIES FOR DIFFERENT METHOD

Methods for generating error patterns	**Number of the errors t**				
	2	***3***	***4***	***5***	***6***
Rank	<1 s	<1 s	<1 s	7 s	10 m
Sub-class	<1 s	<1 s	<1 s	5 s	8 m
Iterative	<1 s	<1 s	<1 s	<1 s	<1 s

We can see from the table. 1 that when the number of errors is below 5, all three methods can generate a corresponding library within one second, however, when the number of errors is surpassed 5, there are emerge some differences. For the rank method, it will take 7 seconds to erect an error pattern library when *t* is 5, and it will take 10 minutes to create another library when *t* is 6. The sub-classified method costs lower time by comparing with the rank method, but it also needs 8 minutes to form a library when *t* is 6. The proposed method has the best result. For all different numbers of *t*, the time spent by the proposed method to construct the same libraries that build by the former two methods is all within one second. It is convinced that the proposed method has a better performance in terms of processing speed when compare with the rank method and subclassified method.

V. CONCLUSION AND FUTURE WORK

A model for forming a library of error patterns based on iterative expansion of the error patterns for the syndrome-norm decoding of iterative codes are proposed in this paper. The proposed method differs from the known rank method and subclassified method by it adopts an iterative forming method to build a library of error patterns. By iterative adding extra one error to the patterns that generated in the former iteration and eliminating redundant error patterns and retaining those representative patterns that have distinct feature vectors. The feature vectors are obtained according to the formula (6). The experiments has proved that the proposed method is a very efficient method, which requires less time to build a same

Pattern Recognition and Information Processing (PRIP'2021) : Proceedings of the 15th International Conference, 21–24 Sept. 2021, Minsk, Belarus. – Minsk : UIIP NASB, 2021. – 246 p. – ISBN 978-985-7198-07-8.

library that formed by the rank method and by the subclassified method because the proposed method cancel to enumerate all the permutation of the possible error situation. The result of the proposed method can be applied in the syndrome-norm method, which is a decoding method based on the library matching.

In the future, we would like to analysis these patterns in the library, and for each pattern, we will construct a corresponding correction method. Then conduct a comprehensive comparison with the famous method. Another interesting direction is explore the performance of the syndrome-norm method.

References

[1] S. Lin and D. J. Costello, "Error Control Coding Fundamentals and Applications Second Edition."

[2] R. M. Pyndiah, "Near-optimum decoding of product codes:block turbo codes," IEEE Transactions on Communications, vol. 46, no. 8, pp. 1003–1010, 1998.

[3] F. Chiaraluce and R. Garello, "Extended Hamming product codes analytical performance evaluation for low error rate applications," IEEE Transactions on Wireless Communications, vol. 3, no. 6, pp. 2353–2361, 2004.

[4] B. Fu and P. Ampadu, On hamming product codes with type-II hybrid ARQ for on-chip interconnects. IEEE Trans Circuits Syst I, Reg Papers, vol. 56, no. 9, pp. 2042–2054, 2009.

[5] V. A. Lipnitski, V. K. Konopelko, The theory of syndrome norms in the permutation decoding action unjammable codes. Doklady BGUIR, vol.1, no. 1, pp. 146–157, 2000.

[6] V. K. Konopelko, A. V. Lipnitski and N. V. Spichekova, Point pattern classification and classical number partition problem. Doklady BGUIR, vol. 51, no. 5, pp. 112–117, 2010.

[7] O. G. Smolyakova, V. K. Konopelko Classification vectors of errors at two-dimensional coding of the information. Doklady BGUIR, vol. 37, no. 7, pp. 19–28, 2008.

[8] P. Elias, "Error-free Coding," in Transactions of the IRE Professional Group on Information Theory, vol. 4, no. 4, pp. 29–37, September 1954, doi: 10.1109/TIT.1954.1057464.

[9] S. Lin and D. J. Costello, Error Control Coding, Prentice Hall, Upper Saddle River, NJ, USA, 2nd edition, 2004.

[10] L. Ping, S. Chan, and K. L. Yeung, "Iterative decoding of multi-dimensional concatenated single parity check codes," in Proceedings of the IEEE International Conference on Communications (ICC '98), vol. 1, pp. 131–135, Atlanta, Ga, USA, June 1998.

Pattern Recognition and Information Processing (PRIP'2021) : Proceedings of the 15th International Conference, 21–24 Sept. 2021, Minsk, Belarus. – Minsk : UIIP NASB, 2021. – 246 p. – ISBN 978-985-7198-07-8.